Star subdivisions and connected even factors in the square of a graph

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joint work with P. Holub, T. Kaiser, L. Xiong, S. Zhang
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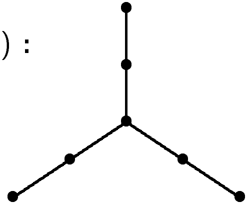
prism over a graph G - Cartesian product $G \square K_2$ of G with the complete graph K_2

Square of a graph

$S(K_{1,3})$:



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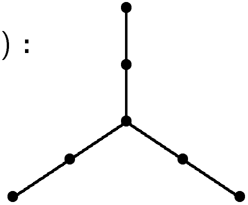

Theorem

[Faudree, Schelp], [Chartrand, Hobbs, Jung, Kapoor, Nash-Williams] *Let G be a 2-connected graph. Then G^2 is hamiltonian connected.*

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Theorem

[El Kadi Abderrezzak, Flandrin, Ryjáček] *If G is a connected graph such that every induced $S(K_{1,3})$ has at least three edges in a block of degree at most 2, then G^2 is hamiltonian.*

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Let s be a positive integer and G be a connected graph such that every induced $S(K_{1,2s+1})$ has at least three edges in a block of degree at most two. Then G^2 has a $[2, 2s]$-factor.

Proofs

Theorem

[Jackson, Ordaz] *Let* $k \geq 2$ *be an integer and* G *a* k*-connected graph. If* $\alpha(G) > k$ *then* $V(G)$ *can be covered with* $\alpha(G) - k$ *disjoint paths.*

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Corollary

Let G be a graph. Then there are at most $\alpha(G)$ disjoint paths covering $V(G)$.

Lemma

Let H be a connected graph and $P = xyz$ a path of length two such that $V(H) \cap V(P) = \{x\}$. If $(HxP)^2$ has a $[2, 2s]$-factor, then one of the following holds:

(a) *H^2 contains a spanning closed s-trail T such that the degree of x in T is at most $2s - 2$, or*

(b) *H^2 contains a spanning s-trail T between x and some $x' \in N_H(x)$.*

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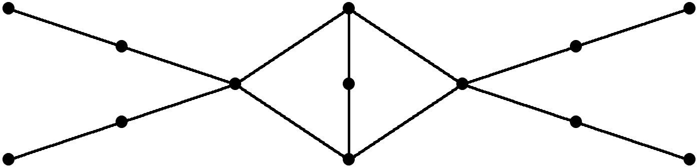
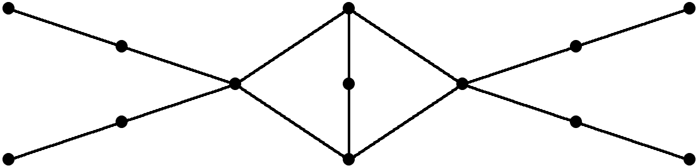
Claim: *Let x be a cut vertex of G and F_1, F_2 two connected subgraphs of G such that F_1, F_2 belong to different branches of G at x. Assume that F_2 is nontrivial at x, i.e., F_2 contains an induced $P_3(x) = xyz$ as a proper induced subgraph. Then the graph $G' = F_1 x P_3(x)$ also satisfies (1).*





Corollary

Let G be a simple connected graph with $\Delta(G) \leq 2s$. Then G^2 has a $[2, 2s]$-factor.

Conclusion

Thank you for your attention.