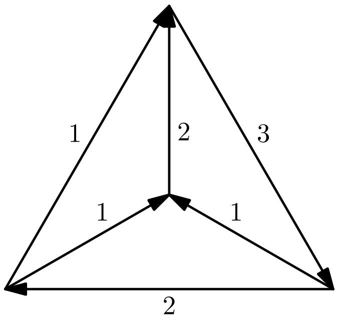
Real flow number of generalized Blanuša snarks

Robert Lukoťka

FMFI UK

Example of a nowhere-zero 4-flow



Real flows on graphs

A real NZ r-flow:

- Orientation
- Function $\varphi : E(G) \to \mathbb{R}$
  - $1 \leq \varphi(e) \leq r - 1$
  - $\sum_{e \in \{v\}^+} \varphi(e) = \sum_{e \in \{v\}^-} \varphi(e)$

Real flow number

- $\Phi_{\mathbb{R}}(G) = \inf\{r \mid G \text{ has a NZ } r\text{-flow}\}$

Basic properties of real NZ flows

- The infimum from the definition is a minimum.
- The real flow number is rational.
- If $\Phi_{\mathbb{R}}(G) = p/q$ then it is sufficient to use values with the denominator q. to create a real NZ p/q-flow.

Graphs with known real flow numbers

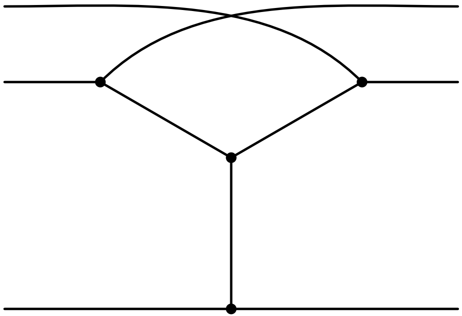
- Complete graphs
- Complete bipartite graphs
- Some "series-parallel graphs"
- Three-edge colourable cubic graphs

Snarks

Objects of our attention:

- Isaacs snarks
- Blanuša snarks
- Goldberg snarks

Isaacs snarks



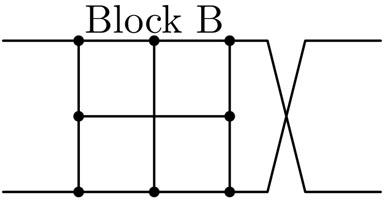
Isaacs snarks

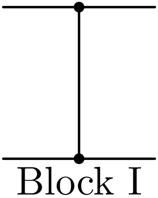
Since Isaacs snark I_k has $8k+4$ vertices:

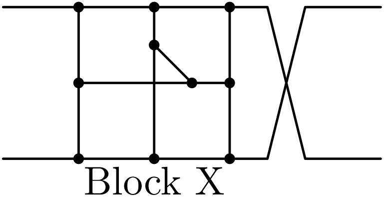
Theorem

The real flow number of the Isaacs snark I_{2k+1} is $\Phi_{\mathbb{R}}(I_{2k+1}) = 4 + 1/k$.

Blanuša snarks







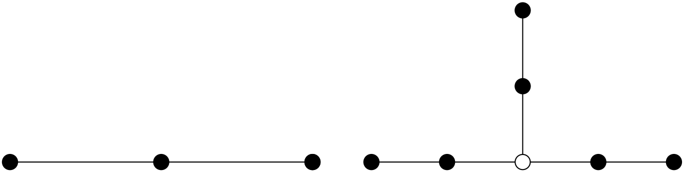
Balanced valuations - simplified

We take:
– a fixed nowhere-zero $(3+\varepsilon)$-flow φ, $\varepsilon < 1/2$,
– a positive orientation O of G.

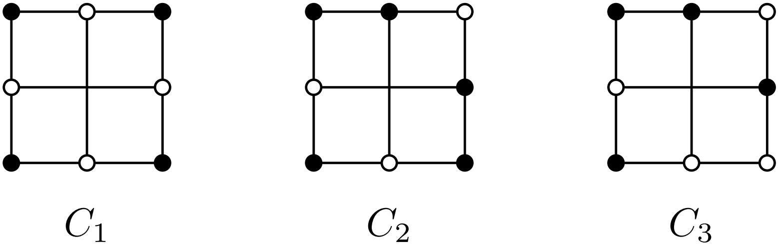
- Two incoming edges – white vertex.
- Two outgoing edges – black vertex.

Number of black vertices = number of white vertices.

Forbidden substructures

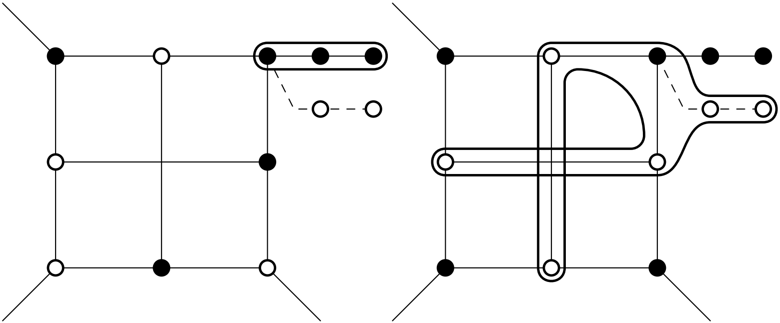


Possible colourings of the basic block



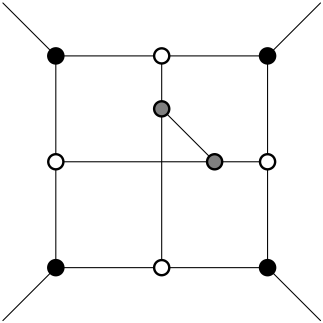
The colouring C_1

The colouring C_1 can not combine with the others:

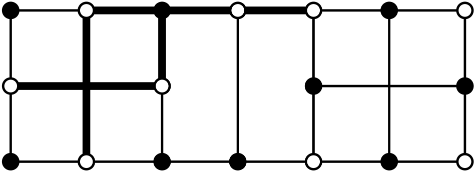


The colouring C_1 is unusable

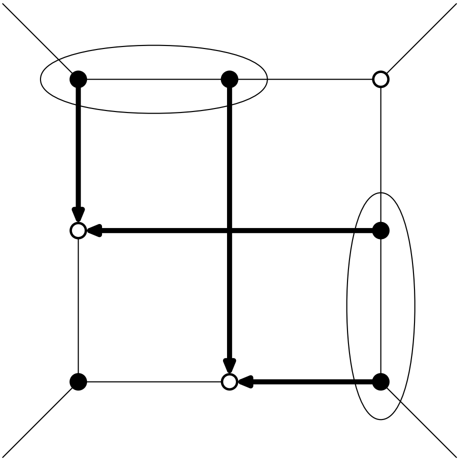
The middle blocks have to be coloured as follows:



The colouring C_1 is unusable



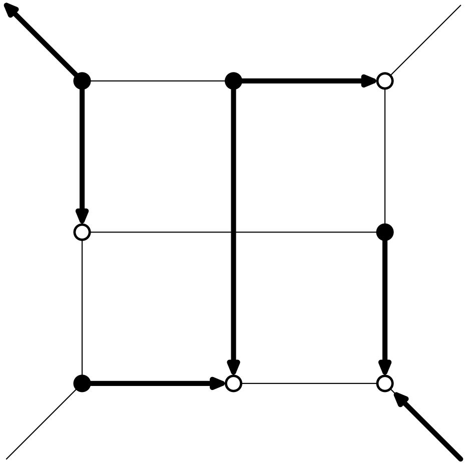
The colouring C_2 is unusable



Tight edges

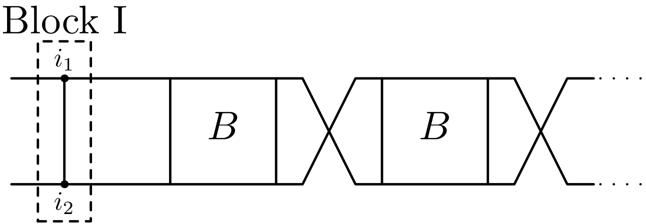
We create a modular flow φ' in $\mathbb{R}/(4+\varepsilon)\mathbb{Z}$ from the flow φ. We take the orientation so that all values are in $\langle 1, 2+\varepsilon/2\rangle$.
Tight edge – an edge with flow value $\langle 1, 1+\varepsilon\rangle$ in φ'.
Semi-tight edge – an edge with flow value $\langle 1, 1+2\varepsilon\rangle$ in φ'.

Structure of tight edges in block B.



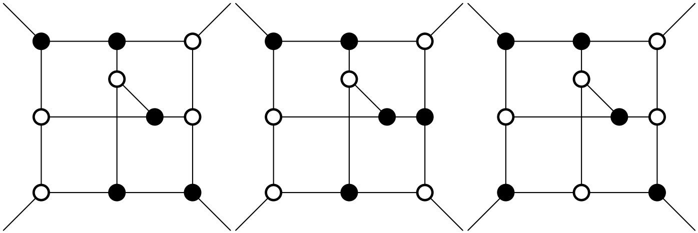
Blanuša snarks of type I

The total flow difference on neighbouring edges of Block I is at most 2ε.



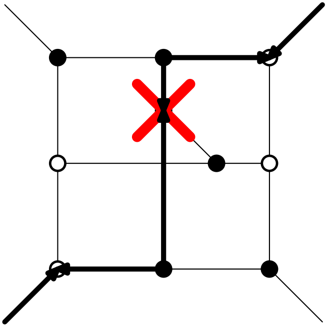
Blanuša snarks of type II

Possible colourings of block X.



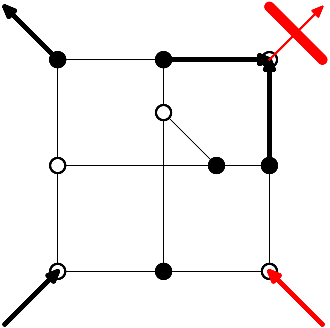
Colouring 1

Two tight edges have incompatible orientation.



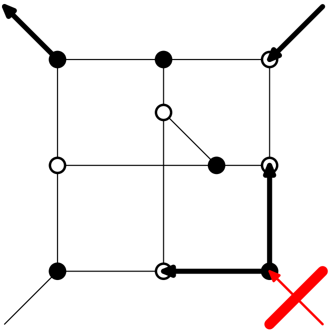
Colouring 2

One of the edges on right side has to be tight – the bottom right one.



The value of the upper right edge is at most $1 + 2\varepsilon$ - impossible.

Colouring 3



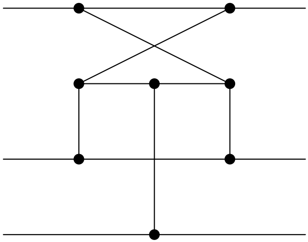
The value of the bottom right edge is at most $1 + 2\varepsilon$ - impossible.

Real flow number of the Blanuša snarks

It is easy to construct 4.5-flows on the Blanuša snarks that are different from the Petersen graph.

- $\Phi_{\mathbb{R}}(B_1^i) = 5.$
- $\Phi_{\mathbb{R}}(B_j^i) = 4 + 1/2,\ j \geq 2.$

Goldberg snarks



Real flow number

Goldberg snark G_{2k+1} has its real flow number

$$4 + 1/(2k+1) \leq \Phi_{\mathbb{R}}(G_{2k+1}) \leq 4 + 1/k.$$

Thank you for your attention.